Equation Word Problems

Equation from a pair of points

Consider the following problem:

1. You fill a glass with 300 ml of water. After sitting for 5 hours, the water has evaporated so that there are only 100 ml left.
   a. Write an equation that describes the amount of water, *w*, in the cup as a function of time, *t*.
   b. How much water was in the cup after 2 hours?
   c. How long did it take for all the water to evaporate?

Overview of the process

1. Identify the quantities involved in the problem.
2. Decide which is the independent variable.
3. Identify at least two points in the problem.
4. Write the equation of the line between those two points.
5. Substitute variable names, if necessary.

Discussion of 1a

Step 1. Identify the quantities involved in the problem

First we read the problem and identify what quantities are involved. In our case, the two quantities are the amount of water and time.

Step 2. Identify the independent variable. (Which quantity causes the other to change?)

The independent variable is the quantity that causes the other one to change. In this case, time causes the amount of water to change (not vice versa), so time, *t*, is the independent variable.

This will make it the *x* in our *x-y* points.

Step 3. Identify at least two x-y coordinates in the problem

We had 100 ml of water at 5 hours and 300 ml of water at, implicitly, 0 hours. These pairs of numbers can be considered the points (0, 100) and (5, 300).

Note that the time is in the x position because it's the independent variable.

Step 4. Write the equation between these two points

The usual process: calculate the slope and then plug one of the points into the template of the slope-intercept form of a line. In this case, you end up with

$$y = -40x + 300$$

Note that the slope of this line represents a rate; in our case, this is the amount of water that evaporates per hour. In other problems, this might be an average speed, an average annual profit, etc.

Step 5. Replace the variable names, if necessary

Since our problem identified the volume of water as *w* and the time as *t*, replace *x* and *y* in the original equation with *w* and *t*.

$$w = -40t + 300$$

Equation from a rate and value

1. A baby Spotted Velociraptor will gain about 3 lbs per day if properly cared for. Hans, your pet velociraptor, is now 2 weeks old and weights 60 lbs. Write an equation for Hans' weight (*w*) as a function of time in days (*d*).

Discussion of 1a

In this case, the problem gives you a rate: 3 lbs. per day; in mathematical terms, this is the slope of the equation describing Hans' weight. This being the case, the problem has given us a slope and a point, (14, 60).

Note that time is once again our *x* value because time is the independent variable: time causes the velociraptor to grow; its growth does *not* cause time to move forward (though it'd be cool if that were so).

Now we can derive the equation relatively easily.

$y - 60 = 3(x - 14)$ ← Point slope form...

$y = 3x + 18$ ← Converted to slope-intercept form

$w = 3d + 18$ ← With proper variable names substituted