Definitions and Theorems

Definitions

Continuous

A function $f(x)$ is *continuous* at $x = c$ iff:

1 $\lim_{x \to c} f(x)$ exists

2 $f(c)$ exists

3 $\lim_{x \to c} f(x) = f(c)$

Differentiable

A function is *differentiable* on open interval (a,b) if $f'(x)$ exists for every value of x on (a,b).

- ▹ Alternative definition: $f(x)$ is differentiable at $x = c$ if $f'(x)$ is continuous at $x = c$.

- ▸ This means that $f(x)$ can't have a cusp or a vertical asymptote on (a,b)

Critical Numbers

If $f(x)$ is defined at $x = c$, then c is a *critical number* of f if one of the following is true:

- ▸ $f'(c) = 0$
  - ▹ *i.e.*, $f(c)$ is a local maximum or minimum
- ▸ $f'(c)$ does not exist.
  - ▹ *i.e.*, $f(c)$ is a discontinuity or a cusp.

Theorems

Intermediate Value Theorem

If f is continuous on $[a,b]$ and k is between $f(a)$ and $f(b)$, then there exists a value c on $[a,b]$ such that

$$f(c) = k$$

Mean Value Theorem

If f is continuous on $[a,b]$ and differentiable on (a,b), then there exists a value c on (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- ▸ *In other words,* there will be some value of x between a and b where the instantaneous slope of the function is equal to the average slope between a and b.

Mean Value Theorem for Integrals

If f is continuous on $[a,b]$, then there exists a value c on $[a,b]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

- *In other words,* there will be some value of x between a and b where the value of the function is equal to the average value of the function between a and b.

Rolle's Theorem

If f is continuous on $[a,b]$ and differentiable on (a,b), and $f(a) = f(\mathrm{b})$, then there exists a value c on (a,b) such that

$$f'(\mathrm{c}) = 0$$

- *In other words,* there will be some value of x between a and b where the function has a local max or local min.

Extreme Values Theorem

If f is continuous on $[a,b]$, then f has both an absolute maximum and absolute minimum on $[a,b]$.