Newton's Method

Description

Newton's Method is a process for approximating the real zeros of a function, *i.e.*, for finding an approximate solution to the equation

$$f(x) = 0$$

The process is iterative; that is, it operates by finding successive approximations to the solution, each approximation closer to the real solution than the last, until you find an approximation that is "close enough."

- "Close enough" usually means the successive approximations are no longer differing from each other by a meaningful amount.
- *e.g.*, perhaps each new approximation is only .0001 different from the previous one.

The Method

- Newton's method starts with an initial guess you must make as to the x-value of the solution.
  - *The actual value of the guess doesn't matter*; it can be as accurate or wildly wrong as you wish, though the closer you are to the real value, the fewer times you'll need to go through the loop.

As a Sequence

- Successive approximations can be expressed as a recursive sequence, as follows:

$$x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$$

Your initial x_n will be the guess that you make at the start of the process.

As a Step-by-Step Process

The above recursive sequence is the equivalent of the following process:

1. Find the derivative of $f(x)$.
2. Make a guess as to what the solution value might be; call this number x_{old}.
3. Calculate a new approximation to the solution using the equation

$$x_{new} = x_{old} + \frac{f(x_{old})}{f'(x_{old})}$$

4. Done or repeat? If your new value is "close enough," you're done; otherwise, go back to step 3.

Organizing Your Work

It's not necessary, but many people find it useful to construct a table with the following headings to keep track of their intermediate steps:

n	x_n	$f(x_n)$	$f'(x_n)$	$f(x_n)/f'(x_n)$